Concept Byte
For Use With Lesson 3-5
ACTIVITY

Graphs in Three Dimensions

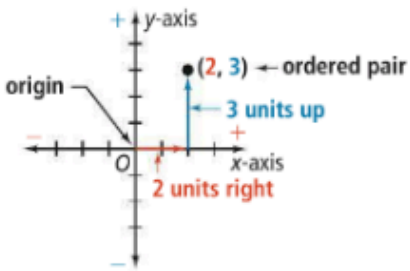
Common Core State Standards

Extends A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MP 4

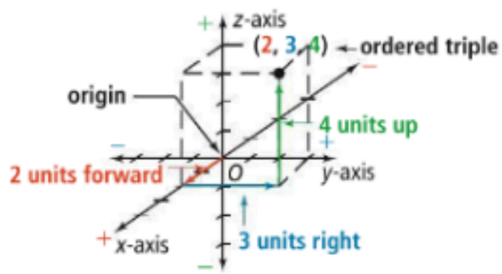
To describe positions in space, you need a three-dimensional coordinate system. You have learned to graph on an *xy*-coordinate plane using ordered pairs. Adding a third axis, the *z*-axis, to the *xy*-coordinate plane creates **coordinate space**. In coordinate space you graph points using **ordered triples** of the form (x, y, z).

Points in a Plane



A two-dimensional coordinate system allows you to graph points in a plane.

Points in Space



A three-dimensional coordinate system allows you to graph points in space.

In the coordinate plane, point (2, 3) is two units right and three units up from the origin. In coordinate space, point (2, 3, 4) is two units forward, three units right, and four units up.

Activity 1

Define one corner of your classroom as the origin of a three-dimensional coordinate system like the classroom shown. Write the coordinates of each item in your coordinate system.

1. each corner of your classroom
2. each corner of your desk
3. one corner of the blackboard
4. the clock
5. the waste-paper basket
6. Pick 3 items in your classroom and write the coordinates of each.

An equation in two variables represents a line in a plane. An equation in three variables represents a plane in space.

Activity 2

Given the following equation in three variables, draw the plane in a coordinate space. $x + 2y - z = 6$

7. Let $x = 0$. Graph the resulting equation in the *yz* plane.

8. Let $y = 0$. Graph the resulting equation in the *xz* plane.

From geometry you know that two lines determine a plane.

9. Sketch the plane $x + 2y - z = 6$.
(If you need help, find a third line by letting $z = 0$ and then graph the resulting equation in the *xy* plane.)

Activity 3

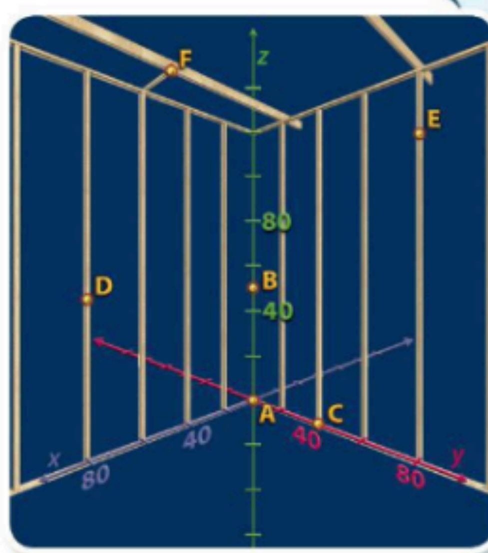
Two equations in three variables represent two planes in space.

10. Draw the two planes determined by the following equations:
$2x + 3y - z = 12$
$2x - 4y + z = 8$

11. Describe the intersection of the two planes above.

Exercises

Find the coordinates of each point in the diagram.



12. *A* **13.** *B* **14.** *C*

15. *D* **16.** *E* **17.** *F*

Sketch the graph of each equation.

18. $x - y - 4z = 8$

19. $x + y + z = 2$

20. $-3x + 5y + 10z = 15$

21. $6x + 6y - 12z = 36$

Graph the following pairs of equations in the same coordinate space and describe their intersection, if any.

22. $-x + 3y + z = 6$
$-3x + 5y - 2z = 60$

23. $-2x - 3y + 5z = 7$
$2x - 3y - 4z = -4$

3-5 Systems With Three Variables

Common Core State Standards

Extends A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

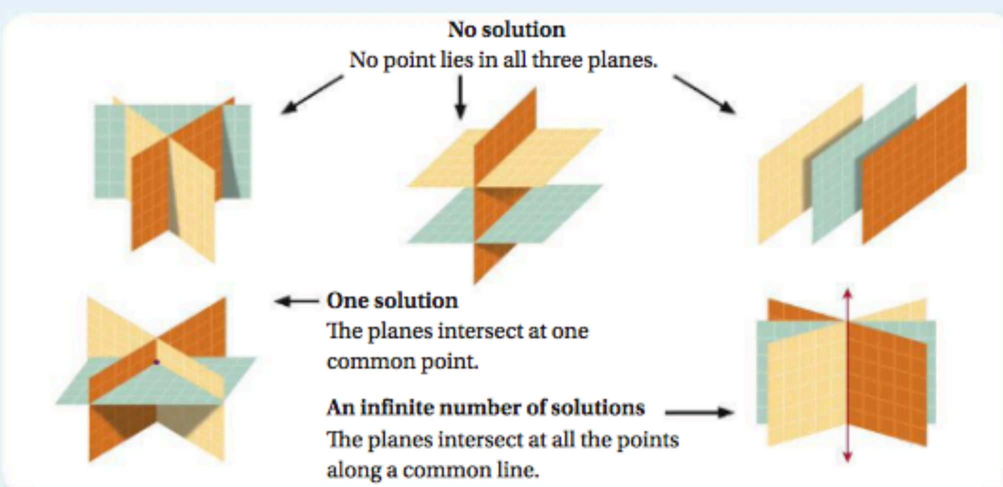
MP 1, MP 3

Objectives To solve systems in three variables using elimination
To solve systems in three variables using substitution

MATHEMATICAL PRACTICES You can represent three relationships involving three unknowns with a system of equations.

Essential Understanding To solve systems of three equations in three variables, you can use some of the same algebraic methods you used to solve systems of two equations in two variables.

You can represent systems of equations in three variables as graphs in three dimensions. The graph of an equation of the form $Ax + By + Cz = D$, where A, B, and C are not all zero, is a plane. You can show the solutions of a three-variable system graphically as the intersection of planes.



You can use the elimination and substitution methods to solve a system of three equations in three variables by working with the equations in pairs. You will use one of the equations *twice*. When one point represents the solution of a system of equations in three variables, write it as an ordered triple (x, y, z).

Problem 1 Solving a System Using Elimination

Think

Which variable do you eliminate first?
Eliminate the variable for which the process requires the fewest steps.

What is the solution of the system? Use elimination. The equations are numbered to make the procedure easy to follow.

$$\begin{array}{l} ①\\ ②\\ ③ \end{array}\begin{cases} 2x - y + z = 4 \\ x + 3y - z = 11 \\ 4x + y - z = 14 \end{cases}$$

Step 1 Pair the equations to eliminate z. Then you will have two equations in x and y.

Add.

$$\begin{array}{ll} ① & 2x - y + z = 4 \\ ② & x + 3y - z = 11 \\ \hline ④ & 3x + 2y = 15 \end{array}$$

Subtract.

$$\begin{array}{ll} ② & x + 3y - z = 11 \\ ③ & 4x + y - z = 14 \\ \hline ⑤ & -3x + 2y = -3 \end{array}$$

Step 2 Write the two new equations as a system. Solve for x and y.

Add and solve for y.

$$\begin{array}{ll} ④ & 3x + 2y = 15 \\ ⑤ & -3x + 2y = -3 \\ \hline & 4y = 12 \\ & y = 3 \end{array}$$

Substitute $y = 3$ and solve for x.

$$\begin{array}{ll} ④ & 3x + 2y = 15 \\ & 3x + 2(3) = 15 \\ & 3x = 9 \\ & x = 3 \end{array}$$

Think

Does it matter which equation you substitute into to find z?
No, you can substitute into any of the original three equations.

Step 3 Solve for z. Substitute the values of x and y into one of the original equations.

①	$2x - y + z = 4$	Use equation ①.
	$2(3) - 3 + z = 4$	Substitute.
	$6 - 3 + z = 4$	Simplify.
	$z = 1$	Solve for z.

Step 4 Write the solution as an ordered triple. The solution is $(3, 3, 1)$.

Got It? 1. What is the solution of the system? Use elimination. Check your answer in all three original equations.

$$\begin{cases} x - y + z = -1 \\ x + y + 3z = -3 \\ 2x - y + 2z = 0 \end{cases}$$

You can apply the method in Problem 1 to most systems of three equations in three variables. You may need to multiply in one, two, or all three equations by one, two, or three nonzero numbers. Your goal is to obtain a system—equivalent to the original system—with coefficients that allow for the easy elimination of variables.

Problem 2 Solving an Equivalent System

What is the solution of the system? Use elimination.

$$\begin{cases} x + y + 2z = 3 & ① \\ 2x + y + 3z = 7 & ② \\ -x - 2y + z = 10 & ③ \end{cases}$$

Think	Write
You are trying to get two equations in *x* and *z*. Multiply ① so you can add it to ② and eliminate *y*. Do the same with ② and ③.	① $\begin{cases} x + y + 2z = 3 \\ 2x + y + 3z = 7 \end{cases}$ ② → $\begin{array}{r} -x - y - 2z = -3 \\ 2x + y + 3z = 7 \\ \hline x + z = 4 \end{array}$ ④ ② $\begin{cases} 2x + y + 3z = 7 \\ -x - 2y + z = 10 \end{cases}$ ③ → $\begin{array}{r} 4x + 2y + 6z = 14 \\ -x - 2y + z = 10 \\ \hline 3x + 7z = 24 \end{array}$ ⑤
Multiply ④ so you can add it to ⑤ and eliminate *x*.	④ $\begin{cases} x + z = 4 \\ 3x + 7z = 24 \end{cases}$ ⑤ → $\begin{array}{r} -3x - 3z = -12 \\ 3x + 7z = 24 \\ \hline 4z = 12 \\ z = 3 \end{array}$
Substitute $z = 3$ into ④. Solve for *x*.	$x + 3 = 4$ $x = 1$
Substitute the values for *x* and *z* into ① to find *y*. Check the answer in the three original equations.	$x + y + 2z = 3$ $1 + y + 2(3) = 3$ $y = -4$ Check $1 + (-4) + 2(3) = 3$ ✔ $2(1) + (-4) + 3(3) = 7$ ✔ $-(1) - 2(-4) + 3 = 10$ ✔ The solution is $(1, -4, 3)$.

Got It? 2. a. What is the solution of the system? Use elimination.

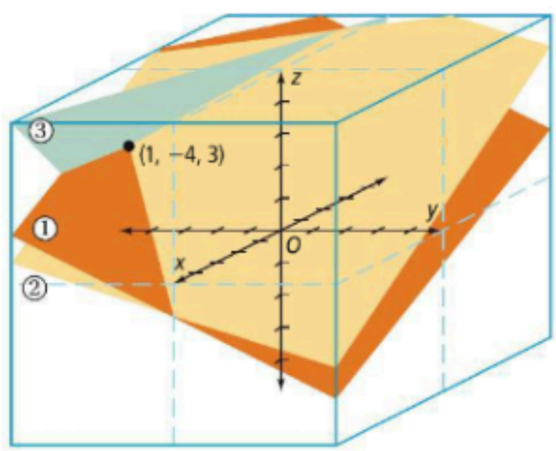
$$\begin{cases} x - 2y + 3z = 12 \\ 2x - y - 2z = 5 \\ 2x + 2y - z = 4 \end{cases}$$

b. Reasoning Could you have used elimination in another way? Explain.

Here is a graphical representation of the solution of Problem 2. The graphs are enclosed in a 10-by-10-by-10 cube with the origin of the coordinate axes at the center.

The graphs of equations ①, ②, and ③, are planes ①, ②, and ③, respectively.

Each pair of planes intersects in a line. The three lines intersect in $(1, -4, 3)$, the solution of the system.



You can also use substitution to solve a system of three equations. Substitution is the best method to use when you can easily solve one of the equations for a single variable.

Think
Which equation should you solve for one of its variables? Look for an equation that has a variable with coefficient 1.

Problem 3 Solving a System Using Substitution

Multiple Choice **What is the x-value in the solution of the system?**

$$\begin{cases} 2x + 3y - 2z = -1 & ① \\ x + 5y = 9 & ② \\ 4z - 5x = 4 & ③ \end{cases}$$

(A) 1 (B) 4 (C) 6 (D) 10

Step 1 Choose equation ②. Solve for x.

② $x + 5y = 9$

$x = 9 - 5y$

Step 2 Substitute the expression for x into equations ① and ③ and simplify.

①
$$\begin{aligned} 2x + 3y - 2z &= -1 \\ 2(9 - 5y) + 3y - 2z &= -1 \\ 18 - 10y + 3y - 2z &= -1 \\ 18 - 7y - 2z &= -1 \end{aligned}$$
④ $-7y - 2z = -19$

③
$$\begin{aligned} 4z - 5x &= 4 \\ 4z - 5(9 - 5y) &= 4 \\ 4z - 45 + 25y &= 4 \\ 4z + 25y &= 49 \end{aligned}$$
⑤ $25y + 4z = 49$

Step 3 Write the two new equations as a system. Solve for y and z.

$$\begin{cases} -7y - 2z = -19 & ④ \\ 25y + 4z = 49 & ⑤ \end{cases}$$

$$\begin{aligned} -14y - 4z &= -38 \quad \text{Multiply by 2.} \\ 25y + 4z &= 49 \quad \text{Then add.} \\ \hline 11y &= 11 \\ y &= 1 \end{aligned}$$

④
$$\begin{aligned} -7y - 2z &= -19 \\ -7(1) - 2z &= -19 \quad \text{Substitute the value of } y \text{ into ④.} \\ -2z &= -12 \\ z &= 6 \end{aligned}$$

Step 4 Use one of the original equations to solve for x.

②
$$\begin{aligned} x + 5y &= 9 \\ x + 5(1) &= 9 \quad \text{Substitute the value of } y \text{ into ②.} \\ x &= 4 \end{aligned}$$

The solution of the system is (4, 1, 6), and $x = 4$.

The correct answer is B.

Got It? **3. a.** What is the solution of the system? Use substitution.

$$\begin{cases} x - 2y + z = -4 \\ -4x + y - 2z = 1 \\ 2x + 2y - z = 10 \end{cases}$$

b. Reasoning In Problem 3, was it necessary to find the value of z to solve the problem? Explain.

Problem 4 Solving a Real-World Problem

Business **You manage a clothing store and budget $6000 to restock 200 shirts. You can buy T-shirts for $12 each, polo shirts for $24 each, and rugby shirts for $36 each. If you want to have twice as many rugby shirts as polo shirts, how many of each type of shirt should you buy?**

Think

How many unknowns are there?
There are three unknowns: the number of each type of shirt.

Relate

T-shirts + polo shirts + rugby shirts = 200

rugby shirts = 2 • polo shirts

12 • T-shirts + 24 • polo shirts + 36 • rugby shirts = 6000

Define

Let x = the number of T-shirts.

Let y = the number of polo shirts.

Let z = the number of rugby shirts.

Write

$$\begin{cases} ① & x + y + z = 200 \\ ② & z = 2 \cdot y \\ ③ & 12 \cdot x + 24 \cdot y + 36 \cdot z = 6000 \end{cases}$$

Step 1 Since 12 is a common factor of all the terms in equation ③, write a simpler equivalent equation.

$$\begin{cases} ③ & 12x + 24y + 36z = 6000 \\ ④ & x + 2y + 3z = 500 \end{cases}$$ Divide by 12.

Step 2 Substitute $2y$ for z in equations ① and ④. Simplify to find equations ⑤ and ⑥.

① $x + y + z = 200$
$x + y + (2y) = 200$
⑤ $x + 3y = 200$

④ $x + 2y + 3z = 500$
$x + 2y + 3(2y) = 500$
⑥ $x + 8y = 500$

Step 3 Write ⑤ and ⑥ as a system. Solve for x and y.

$$\begin{cases} ⑤ & x + 3y = 200 \\ ⑥ & x + 8y = 500 \end{cases}$$

$-x - 3y = -200$ Multiply by -1.
$x + 8y = 500$ Then add.
$5y = 300$
$y = 60$

⑤ $x + 3y = 200$
$x + 3(60) = 200$ Substitute the value of y into ⑤.
$x = 20$

Step 4 Substitute the value of y in ② and solve for z.

② $z = 2y$
$z = 2(60) = 120$

You should buy 20 T-shirts, 60 polo shirts, and 120 rugby shirts.

Got It? **4.** Suppose you want to have the same number of T-shirts as polo shirts. Buying 200 shirts with a budget of $5400, how many of each shirt should you buy?

Lesson Check

Do you know HOW?

Solve each system.

1. $\begin{cases} 2y - 3z = 0 \\ x + 3y = -4 \\ 3x + 4y = 3 \end{cases}$

2. $\begin{cases} 3x + y - 2z = 22 \\ x + 5y + z = 4 \\ x = -3z \end{cases}$

3. $\begin{cases} 2x + 3y - 2z = 1 \\ -x - y + 2z = 5 \\ 3x + 2y - 3z = -6 \end{cases}$

4. $\begin{cases} 2x - y + z = -2 \\ x + 3y - z = 10 \\ x + 2z = -8 \end{cases}$

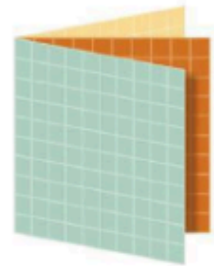
Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Reasoning** How do you decide whether substitution is the best method to solve a system in three variables?

6. **Error Analysis** A classmate says that the system consisting of $x = 0$, $y = 0$, and $z = 0$ has no solution. Explain the student's error.

7. **Writing** How many solutions does this system have? Explain your answer in terms of intersecting planes. (*Hint:* Is the system dependent? inconsistent?)

$\begin{cases} ① & 2x - 3y + z = 5 \\ ② & 2x - 3y + z = -2 \\ ③ & -4x + 6y - 2z = 10 \end{cases}$



8. The graph of a system is shown. How many solutions does this system have? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Solve each system by elimination. Check your answers.

See Problems 1 and 2.

9. $\begin{cases} x - y + z = -1 \\ x + y + 3z = -3 \\ 2x - y + 2z = 0 \end{cases}$

10. $\begin{cases} x - y - 2z = 4 \\ -x + 2y + z = 1 \\ -x + y - 3z = 11 \end{cases}$

11. $\begin{cases} -2x + y - z = 2 \\ -x - 3y + z = -10 \\ 3x + 6z = -24 \end{cases}$

12. $\begin{cases} a + b + c = -3 \\ 3b - c = 4 \\ 2a - b - 2c = -5 \end{cases}$

13. $\begin{cases} 6q - r + 2s = 8 \\ 2q + 3r - s = -9 \\ 4q + 2r + 5s = 1 \end{cases}$

14. $\begin{cases} x - y + 2z = -7 \\ y + z = 1 \\ x = 2y + 3z \end{cases}$

15. $\begin{cases} 3x + 3y + 6z = 9 \\ 2x + y + 3z = 7 \\ x + 2y - z = -10 \end{cases}$

16. $\begin{cases} 3x - y + z = 3 \\ x + y + 2z = 4 \\ x + 2y + z = 4 \end{cases}$

17. $\begin{cases} x - 2y + 3z = 12 \\ 2x - y - 2z = 5 \\ 2x + 2y - z = 4 \end{cases}$

18. $\begin{cases} x + 2y = 2 \\ 2x + 3y - z = -9 \\ 4x + 2y + 5z = 1 \end{cases}$

19. $\begin{cases} 3x + 2y + 2z = -2 \\ 2x + y - z = -2 \\ x - 3y + z = 0 \end{cases}$

20. $\begin{cases} x + 4y - 5z = -7 \\ 3x + 2y + 3z = 7 \\ 2x + y + 5z = 8 \end{cases}$

Solve each system by substitution. Check your answers.

See Problems 3 and 4.

21. $\begin{cases} x + 2y + 3z = 6 \\ y + 2z = 0 \\ z = 2 \end{cases}$

22. $\begin{cases} 3a + b + c = 7 \\ a + 3b - c = 13 \\ b = 2a - 1 \end{cases}$

23. $\begin{cases} 5r - 4s - 3t = 3 \\ t = s + r \\ r = 3s + 1 \end{cases}$

24. $\begin{cases} 13 = 3x - y \\ 4y - 3x + 2z = -3 \\ z = 2x - 4y \end{cases}$

25. $\begin{cases} x + 3y - z = -4 \\ 2x - y + 2z = 13 \\ 3x - 2y - z = -9 \end{cases}$

26. $\begin{cases} x - 4y + z = 6 \\ 2x + 5y - z = 7 \\ 2x - y - z = 1 \end{cases}$

27. $\begin{cases} x - y + 2z = 7 \\ 2x + y + z = 8 \\ x - z = 5 \end{cases}$

28. $\begin{cases} x + y + z = 2 \\ x + 2z = 5 \\ 2x + y - z = -1 \end{cases}$

29. $\begin{cases} 5x - y + z = 4 \\ x + 2y - z = 5 \\ 2x + 3y - 3z = 5 \end{cases}$

STEM **30. Manufacturing** In a factory there are three machines, *A*, *B*, and *C*. When all three machines are working, they produce 287 bolts per hour. When only machines *A* and *C* are working, they produce 197 bolts per hour. When only machines *A* and *B* are working, they produce 202 bolts per hour. How many bolts can each machine produce per hour?

B Apply

31. Think About a Plan In triangle *PQR*, the measure of angle *Q* is three times the measure of angle *P*. The measure of angle *R* is 20 ∘ more than the measure of angle *P*. Find the measure of each angle.

- What are the unknowns in this problem?
- What system of equations represents this situation?
- Which method of solving looks easier for this problem?

32. Sports A stadium has 49,000 seats. Seats sell for \$25 in Section A, \$20 in Section B, and \$15 in Section C. The number of seats in Section A equals the total number of seats in Sections B and C. Suppose the stadium takes in \$1,052,000 from each sold-out event. How many seats does each section hold?

Solve each system using any method.

33. $\begin{cases} x - 3y + 2z = 11 \\ -x + 4y + 3z = 5 \\ 2x - 2y - 4z = 2 \end{cases}$

34. $\begin{cases} x + 2y + z = 4 \\ 2x - y + 4z = -8 \\ -3x + y - 2z = -1 \end{cases}$

35. $\begin{cases} 4x - y + 2z = -6 \\ -2x + 3y - z = 8 \\ 2y + 3z = -5 \end{cases}$

36. $\begin{cases} 4a + 2b + c = 2 \\ 5a - 3b + 2c = 17 \\ a - 5b = 3 \end{cases}$

37. $\begin{cases} 4x - 2y + 5z = 6 \\ 3x + 3y + 8z = 4 \\ x - 5y - 3z = 5 \end{cases}$

38. $\begin{cases} 2\ell + 2w + h = 72 \\ \ell = 3w \\ h = 2w \end{cases}$

39. $\begin{cases} 6x + y - 4z = -8 \\ \frac{y}{4} - \frac{z}{6} = 0 \\ 2x - z = -2 \end{cases}$

40. $\begin{cases} 4y + 2x = 6 - 3z \\ x + z - 2y = -5 \\ x - 2z = 3y - 7 \end{cases}$

41. $\begin{cases} 4x - y + z = -5 \\ -x + y - z = 5 \\ 2x - z - 1 = y \end{cases}$

42. Finance A worker received a \$10,000 bonus and decided to split it among three different accounts. He placed part in a savings account paying 4.5% per year, twice as much in government bonds paying 5%, and the rest in a mutual fund that returned 4%. His income from these investments after one year was \$455. How much did the worker place in each account?

Challenge

43. **Open-Ended** Write your own system with three variables. Begin by choosing the solution. Then write three equations that are true for your solution. Use elimination to solve the system.

44. **Geometry** Refer to the regular five-pointed star at the right. Write and solve a system of three equations to find the measure of each labeled angle.

45. **Geometry** In the regular polyhedron described below, all faces are congruent polygons. Use a system of three linear equations to find the numbers of vertices, edges, and faces.

Every face has five edges and every edge is shared by two faces. Every face has five vertices and every vertex is shared by three faces. The sum of the number of vertices and faces is two more than the number of edges.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 4

In the Apply What You've Learned section in Lesson 3-2, you represented some of the criteria given on page 133 using equations in three variables. Now, you will create another equation needed to solve the problem. Choose from the following words and equations to complete the sentences below.

multiply	substitution	three
a graph	divide	two
$\frac{x}{3} + \frac{y}{20} + \frac{z}{10} = 2$	elimination	$x + y + z = 28$

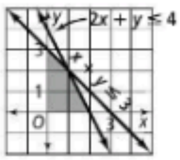
a. If a distance traveled is given in *miles*, then you should __?__ by the rate in *miles per hour* to find the time traveled in *hours*.

b. To create a system of equations in three variables that you can solve, you need to write at least __?__ equations.

c. The equation __?__ models the time in hours each section of the triathlon takes an elite athlete to complete.

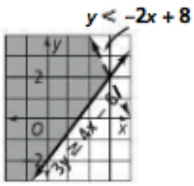
25. C

27.



vertices: (0, 0), (2, 0), (0, 3), (1, 2)

28.



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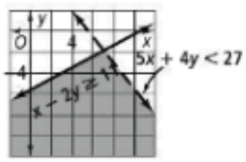
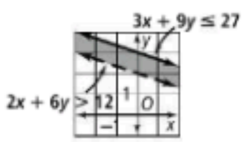


30.



31. 1 **32.** −34 **33.** 24 **34.** 65 **35.** (0, 6), (−3, 0) **36.** (0, 4), (18, 0) **37.** (0, −1), (1, 0)

Lesson 3-5 pp. 166–173

Got It? 1. (4, 2, −3) **2. a.** (4, −1, 2) **b.** Answers may vary. Sample: Yes; you can choose to eliminate either *x*, *y*, or z resulting in a system of equations in 2 variables. **3. a.** (2, 1, −4) **b.** No; in Step 1 we solved for *x* in terms of *y* only. Therefore, once we found the value of *y* we could have substituted that value into the equation we wrote in Step 1 and solved for *x* without ever finding the *z*-value. **4.** 50 T-shirts, 50 polo shirts, and 100 rugby shirts
Lesson Check 1. (5, −3, −2) **2.** (6, 0, −2) **3.** (0, 3, 4) **4.** (2, 1, −5) **5.** Answers may vary. Sample: Substitution is the best method to use when one of the equations can be solved easily for one variable. **6.** Answers may vary. Sample: (0, 0, 0) is a unique solution to a system of three variables. The planes intersect at one common point. When a system has no solution, no point lies in all three planes. **7.** No solution since no point lies in all three planes. The three planes are parallel. **8.** infinitely many solutions
Exercises 9. (4, 2, −3) **11.** (2, 1, −5) **13.** $\left(\frac{1}{2}, -3, 1\right)$ **15.** (1, −4, 3) **17.** (4, −1, 2) **19.** $\left(-\frac{10}{13}, -\frac{2}{13}, \frac{4}{13}\right)$ **21.** (8, −4, 2) **23.** (−2, −1, −3) **25.** (0, 1, 7) **27.** (5, −2, 0) **29.** (1, 3, 2) **31.** $m\angle P = 32°$; $m\angle Q = 96°$; $m\angle R = 52°$ **33.** (8, 1, 3) **35.** $\left(\frac{1}{2}, 2, -3\right)$ **37.** no solution **39.** (2, 4, 6) **41.** (0, 2, −3)
43. Answers may vary. Sample: Solution is (1, 2, 3).

$$\begin{cases} x + y + z = 6 \\ 2x - y + 2z = 6 \\ 3x + 3y + z = 12 \end{cases}$$

45. Let *E*, *F*, and *V* represent the number of edges, faces, and vertices, respectively. From the first statement, $E = \frac{5}{2}F$. From the second statement, $V = \frac{5}{3}F$. From the third statement, $V + F = E + 2$. Solving this system of 3 equations yields $E = 30$, $F = 12$, and $V = 20$.

Lesson 3-6 pp. 174–181

Got It? 1. 17

2. a. $\left[\begin{array}{cc|c} -4 & -2 & 7 \\ 3 & 1 & -5 \end{array}\right]$ **b.** $\left[\begin{array}{ccc|c} 4 & -1 & 2 & 1 \\ 0 & 1 & 5 & 20 \\ 2 & 1 & 0 & 7 \end{array}\right]$

3. $\begin{cases} 2x = 6 \\ 5x - 2y = 1 \end{cases}$

4. a. (1, 2) **b.** elimination; you use the same steps to solve **5.** $\left(1, \frac{1}{2}, 3\right)$

Lesson Check 1. 2 × 1 **2.** 2 × 4

3. $\left[\begin{array}{cc|c} 3 & 5 & 0 \\ 1 & 1 & 2 \end{array}\right]$ **4.** $\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 1 & 0 & 2 & 8 \\ 0 & 2 & -1 & 1 \end{array}\right]$

5. 16 **6.** a_{21} is 0, the element in row 2, column 1. a_{12} is −9, the element in row 1 and column 2. **7.** Answers may vary. Sample: The entry fee to a school play is $2 for adults. Jamie paid a total of $8 for 4 student entry fees and 2 adult entry fees. What is the student entry fee?
Exercises 9. 1 **11.** 8

13. $\left[\begin{array}{cc|c} 3 & 2 & 16 \\ 0 & 1 & 5 \end{array}\right]$ **15.** $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 150 \\ 2 & 0 & 1 & 425 \\ 0 & 1 & 3 & 0 \end{array}\right]$

17. $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & -2 & -1 & 5 \\ 2 & -1 & 2 & 8 \end{array}\right]$ **19.** $\begin{cases} 5x + y = -3 \\ -2x + 2y = 4 \end{cases}$

21. $\begin{cases} 2x + y + z = 1 \\ x + y + z = 2 \\ x - y + z = -2 \end{cases}$ **23.** $\begin{cases} 5x + 2y + z = 5 \\ 4x + y + 2z = 8 \\ x + 3y - 6z = 2 \end{cases}$

25. (−1, 0) **27.** (4, 6) **29.** (2, 3)

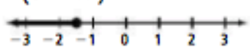
31. $10,000 at 4% and $15,000 at 6%;
Let x = amount invested at 4% and
y = amount invested at 6%.

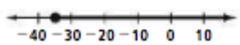
$$\begin{cases} x + y = 25{,}000 \\ 0.04x + 0.06y = 1300 \end{cases}$$

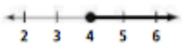
$$\left[\begin{array}{cc|c} 1 & 1 & 25000 \\ 0.04 & 0.06 & 1300 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 10000 \\ 0 & 1 & 15000 \end{array}\right]$$

33. (3, 1,1) **35.** (35, −22, −16) **37.** (1, 1, 1, 1)
39. (2, 3) **41.** 1 qt. of red paint: $7.75; 1 qt. of yellow paint: $5.75 **43.** Answers may vary. Sample: 0; 0

45. (8, 2) **47.** $\left(\frac{1}{8}, -\frac{1}{17}\right)$ **49.** G

51. $x \le -\frac{3}{2}$;


52. $x \ge -35$;


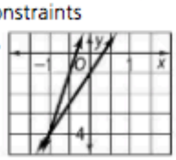
53. $x \ge 4$;


54. $\frac{15}{2}, -\frac{9}{2}$ **55.** 10, −10 **56.** 10, −6 **57.** $y = 2x$

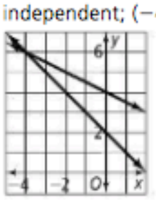
58. $y = \frac{1}{3}x$

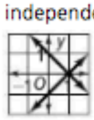
Chapter Review pp. 183–186

1. independent system **2.** Linear programming; constraints

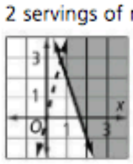
3.

independent; (−1, −4)

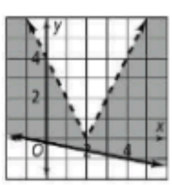
4. dependent **5.** inconsistent **6.** dependent

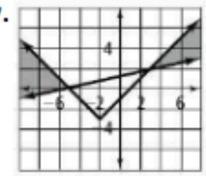
7. independent; (−4, 6)


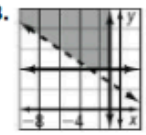
8. independent; (1, 0)


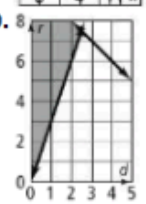
9. 3 pens **10.** (−1, −2) **11.** (0, −5) **12.** (−2, 3)
13. inconsistent; no solution **14.** 1 serving of roast beef and 2 servings of mashed potatoes

15.


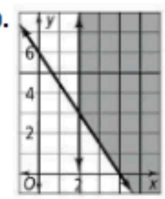
16.


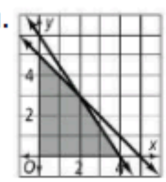
17.


18.


19.

Let r = amount of regular coffee and d = amount of decaffeinated coffee

$$\begin{cases} r + d \le 10 \\ r \ge 3d,\ r \ge 0,\ d \ge 0 \end{cases}$$

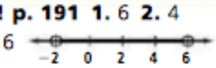
20.

vertices: (4, 0) and (2, 3);
$C = 4$ is minimized at (4, 0).

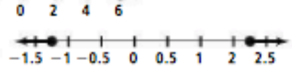
21.

vertices: (0, 0), (4, 0), (2, 3), (0, 5);
$P = 25$ is maximized at (0, 5).

22. 50 chef's salads and 50 Caesar salads
23. (1, 3, −2) **24.** (−4, 1, −5) **25.** (6, 0, −2)
26. no solution **27.** $\left(\frac{1}{2}, \frac{1}{4}\right)$ **28.** (1, −1)
29. (2, −4, 6) **30.** (5, 2, −3)

Chapter 4

Get Ready! p. 191 **1.** 6 **2.** 4

3. $-2 < x < 6$


4. $y \le -\frac{5}{4}$ or $y \ge \frac{9}{4}$


5. $y = 9x - 10$